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A note on the singularity theorem for supergravity SD-branes

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Abstract: Recently the cosmological singularity theorems were shown to exclude non-singular full SD-brane gravitational backgrounds. We comment on the relation between this and previous work as well as provide a more geometric formulation. We point out that some setups of physical interest escape these theorems: cosmological applications, half-SDp-branes and decaying unstable Dp-branes for general p. We also provide indications that the space-filling full SD8-brane (in $d = 10$) escapes as well, because of the important rôle of Ramond-Ramond fields. In any case, tachyon cosmology is not ruled out by the no-go theorem. Lastly, we remark upon interesting directions for potential generalizations of the theorem, and quantum corrections.

Keywords: p-branes, D-branes, Tachyon Condensation.

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Contents

1. Introduction

SD-branes, introduced in [2], are objects of considerable physical interest. There have been a number of works elucidating their physical properties. Some investigations were done using CFT methods, i.e., including stringy α' corrections but no g_s loop effects or gravitational backreaction [3]–[5]. Others were focused on the problem of finding the supergravity fields exerted by a large number of coincident SD-branes [6]–[10].

SD-branes are not BPS, and they are not stable either. It is not *a priori* clear whether there should exist a totally smooth supergravity description of these objects, i.e., solutions without singularities. Several groups have investigated this question at various levels of approximation. Initial works included only the bulk supergravity fields in the analysis [6, 7]. Later works [8, 10] concentrated on the rôle of the homogeneous tachyon, the most relevant open-string degree of freedom in the problem. The hope was that including the backreaction between the rolling tachyon and the bulk fields might resolve the singularities previously found. In this analysis, the tachyon starts at or near the top of its potential hill, and subsequently rolls down the hill sending its energy into bulk supergravity modes.

One of those recent papers [10], by the current set of authors, exhibited numerical results indicating singularity-free solutions. This was however not the definitive word on the subject, for several reasons, one being the inability to handle a sufficiently general ansatz for the bulk fields and tachyon. Another reason was a numerical code instability. One source of numerical instability was understood and discussed - it happened even for the tachyon in flat space. However, it turns out that there was another more important source which was not recognized at that time.

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Most recently, a very interesting singularity theorem for full SD-branes was given in [1]. This work showed *analytically* that *full* SDp-branes with spatial codimension one or greater *must* develop singularities either in the past or the future of $t = 0$. This theorem applies to solutions of the type studied in [10], and provides significant progress in the general problem of finding SD-brane supergravity solutions.

We begin this note by briefly reviewing the singularity theorem itself, and giving a complementary formulation. Using it, we show precisely where the numerical code of [10] went astray. The upshot is that the singularity occurred in the past of $t = 0$. This is an inescapable conclusion, given the fact that at least one Hubble parameter in the problem *must* have a nonzero (small) derivative at $t = 0$ in order to satisfy the constraint equation. We then move to showing two situations of notable interest where the no-go theorem does *not* hold: cosmological applications and decaying unstable branes. We also give strong indications that space-filling SD8-branes may escape the no-go theorem completely. In the discussion, we remark on what may resolve the singularities. We also point out several directions in which the no-go theorem could usefully be generalized. One of them is relaxing the restrictiveness of the ansatz for the bulk and tachyon fields; another is allowing tachyon inhomogeneity.

2. The no-go theorem for full SD-branes

There can be two types of open string tachyon evolutions that could correspond to full-SD-branes:

- The tachyon evolves from one side of the potential $V(T)$ ($T \to \pm\infty$) up and over to the other side corresponding to $T \to \mp\infty$;
- the tachyon evolves from one side of the potential ($T \to \pm\infty$) but never reaches the maximum at $T = 0$. There is then a turning point ($\dot{T} = 0$) for some $|T| > 0$ and the future evolution of the SD-brane proceeds towards $T \to \pm\infty$.

The singularity theorem of [1] clearly states that the corresponding tachyon evolutions will *always* lead to a curvature singularity either in the past (big-bang) or the future (big-crunch) depending on the boundary conditions on the tachyon and the supergravity fields. This theorem assumes an homogeneous tachyon field, $T = T(t)$, and a metric ansatz of the form

$$ds^2 = -dt^2 + a_{||}^2(t)dx_{||}^2 + a_{\perp}^2(t)dx_{\perp}^2\,, \tag{2.1}$$

where $dx_{||}^2$ and $dx_{\perp}^2$ are maximally symmetric spaces of flat ($k_{||} = 0$ or $k_{\perp} = 0$), positive ($k_{||} = 1$ or $k_{\perp} = 1$) or negative ($k_{||} = -1$ or $k_{\perp} = -1$) curvature. Each constant time Cauchy surface of the SD-brane is associated with the volume function

$$V_{\rm S} = v\, a_{||}^{p+1} a_{\perp}^{8-p} = v\, a^9\,, \tag{2.2}$$

where v is a constant. The systems under consideration are either type-IIa or type-IIb supergravity. As described in [8, 10] the setup consists in an unstable brane source (the instability being driven by the corresponding open string tachyon) coupled to the massless supergravity fields: graviton, dilaton and Ramond-Ramond field. The ansatz for the dilaton and the Ramond-Ramond field strength are respectively

$$\Phi = \Phi(t)\,, \qquad \text{and} \qquad F_{tx_1\ldots x_{p+1}} = A(t) a_{\|}^{p+1}\,. \tag{2.3}$$

The theorem in [1] relies on the following combination of the Einstein equations,

$$9\frac{\ddot{a}}{a} = -\left[I_{\rm sugra} + I_{\rm tachyon}\right], \tag{2.4}$$

where

$$I_{\rm sugra} = \frac{1}{4}\left[2\dot{\Phi}^2 + \frac{(7-p)}{4}e^{\Phi(3-p)/2}A^2\right] + \frac{(p+1)(8-p)}{9}\left(\frac{\dot{a}_{\|}}{a_{\|}} - \frac{\dot{a}_{\perp}}{a_{\perp}}\right)^2, \tag{2.5}$$

$$I_{\rm tachyon} = \frac{\lambda V(T) e^{\Phi\left(\frac{p}{4}-\frac{1}{2}\right)}}{16 a_{\perp}^{8-p}}\left(\frac{7}{\sqrt{\Delta}} - (p+1)\sqrt{\Delta}\right), \tag{2.6}$$

with $\Delta \equiv 1 - e^{-\Phi/2}\dot{T}^2$ and λ is a constant quantifying the backreaction between the rolling tachyon and bulk fields (see [10] for a definition). Typically we find that the quantity Δ starts out at (or near) unity at $t = 0$, and becomes monotonically smaller with time.

The singularity theorem of [1] states that for full-SD-brane evolution a singularity will develop unless the gravitational sources introduced provide enough leeway that $\ddot{a}/a$ can become positive. Clearly, $I_{\rm sugra} \geq 0$ so in the context of pure supergravity (no explicit introduction of extended sources) the volume of a full-SD-brane will never experience a period of positive acceleration. This should be related with failed earlier attempts at finding non-singular SD-brane solutions in the context of supergravity [2, 7, 6]. The hope was [8, 10] that the tachyon might be able to resolve this singularity. It can be easily shown that for $p < 7$ we have $I_{\rm tachyon} \geq 0$ which implies that introducing a tachyon source cannot be used to circumvent the singularity theorem.

The most general supergravity solutions obtained in [7] were associated with a world-volume which is anisotropic while the ansatz used to derive the singularity theorem in [1] is isotropic. Only in the anisotropic case were there solutions free of curvature singularities. Presumably the anisotropy modifies the r.h.s. of eq. (2.4) allowing the volume to gain positive acceleration during its evolution in such a way as to avoid big-crunch or big-bang singularities. Also, tachyon inhomogeneity, and any effect on the bulk fields' ansatz, was not handled in the singularity theorem either.

However, for $p = 8$, $I_{\rm tachyon}$ can be negative[1] which means the tachyon can drive the volume of the SD-brane into periods of positive acceleration therefore potentially avoiding the big-crunch or big-bang singularities predicted by the theorem. We provide some remarks on this specific case in the next section.

[1]For $p = 7$ $I_{\rm tachyon}$ can also be negative but [1] showed that there is nevertheless a singularity theorem in this case.

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In short, the singularity theorem implies that full-SD-branes are singular in one of two ways. Type I: full-SD-branes will evolve out of a big-bang singularity (the singularity is in the past of the tachyon evolution) or, type II: full SD-branes lead to a big-crunch singularity in the future. The results of [1] clearly show that during the evolution of the tachyon the Hubble function

$$H = \frac{(p+1)}{9} H_{||} + \frac{(8-p)}{9} H_{\perp} \,, \tag{2.7}$$

where $H_{||} = \dot{a}_{||}/a$ and $H_{\perp} = \dot{a}_{\perp}/a$, will diverge in finite time. Inspection of the associated Ricci scalar expression

$$\begin{aligned} \mathcal{R}(t) \;=\; & -2(p+1)(8-p)H_{||}H_{\perp} - p(p+1)\left[H_{||}^2 + \frac{k_{||}}{a_{||}^2}\right] - (8-p)(7-p)\left[H_{\perp}^2 + \frac{k_{\perp}}{a_{\perp}^2}\right] + \\ & + \frac{1}{4}\left[(p+1)(p-7)P_{||} - (8-p)P_{\perp} + 8(p+1)\rho\right] \end{aligned} \tag{2.8}$$

shows that, unless there is an unlikely conspiracy among terms, this corresponds to a curvature singularity. In the above expression, ρ is the energy density, $P_{||}$ the pressure in the unstable brane worldvolume, and $P_{\perp}$ the pressure in the transverse directions. (Details are in [1, table 1] and will be shown here only as needed.)

At this stage we would like to reformulate the singularity theorem of [1] in a perhaps more geometric form. Depending on the intrinsic geometry of the SD-brane of interest, i.e., the value of $k_{||}$ and $k_{\perp}$, the constant of proportionality v in (2.2) will vary but it is irrelevant for the following analysis. Let us consider in details the cases for which $k_{||} = 0$ and $k_{\perp} = -1$. Asymptotically flat boundary conditions for a full-SD-brane would then be of the form

$$\begin{aligned} &\lim_{t\to\pm\infty} a_{||}(t) \;=\; a_{\pm\infty} \,, && \lim_{t\to\pm\infty} \dot{a}_{||}(t) \;=\; 0 \,, \\ &\lim_{t\to\pm\infty} a_{\perp}(t) \;=\; \pm(t + \kappa_{\pm\infty}) \,, && \lim_{t\to\pm\infty} \dot{a}_{\perp}(t) \;=\; \pm 1 \,, \end{aligned} \tag{2.9}$$

where $a_{\pm\infty}$ and $\kappa_{\pm\infty}$ are non-zero constants. We do not consider the boundary condition of the type

$$\lim_{t\to\pm\infty} a_{\perp} = +t \tag{2.10}$$

since this will most likely lead to a curvature singularity as $a_{\perp} \to 0$. The average scale factor $a(t)$ associated with the boundary conditions (2.9) behaves like

$$\lim_{t\to\pm\infty} a(t) \sim |t + k_{\pm\infty}|^{\frac{8-p}{9}} \,. \tag{2.11}$$

Figure 1 shows, for $p = 2$, $p = 4$ and $p = 6$, the example of a functions $a(t)$ with this asymptotic behavior. The region $t = 0$ could either correspond to the tachyon reaching the top of the potential (type-I full-SD-branes) or to a turning point (type-II full-SD-branes). For $t > 0$ we have $\ddot{a}/a < 0$, which is consistent with the equations of motion, and for $t < 0$ there is necessarily a region for which $\ddot{a}/a > 0$. Recall that the time-reversal symmetric solutions were excluded from the start for consistency reasons. The important point here is that a behavior as shown on figure 1 can only be obtained if there is a region in the

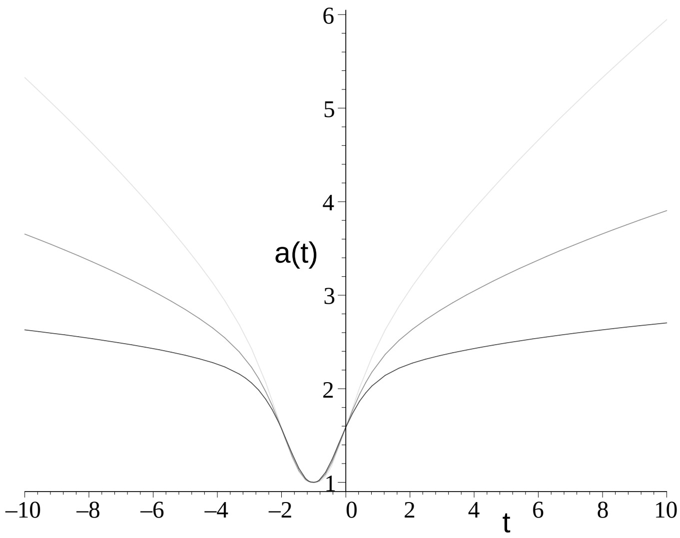

Figure 1: Examples of functions $a(t)$ that *would* be associated with full-SD-branes with $k_{||} = 0$ and $k_\perp = -1$. From top to bottom these correspond to $p = 2$, $p = 4$ and $p = 6$.

evolution of $a(t)$ such that $\ddot{a}/a > 0$. In other words, there is no solutions unless the volume of the full-SD-brane is allowed to go through a phase of positive acceleration. We have seen before that this cannot be accomplished in pure supergravity and that adding the most obvious open string mode, the tachyon, does not help for $p \leq 7$. Presumably, adding more exotic matter or using a more general ansatz, as suggested in [1], could be useful. The results in [7] suggest that allowing for sources inducing worldvolume anisotropies is likely to provide positive results for full-SD-branes.

It therefore seems likely that full-SD-brane solutions with an homogeneous and isotropic worldvolume simply do not exist in pure supergravity and also when the extended sources are associated with a tachyon with dynamics governed by a DBI-like action. This conclusion is of course restricted to the specific ansatz for the unstable branes density in the space transverse to their worldvolume, namely, they were smeared along the corresponding directions. It will be interesting to consider whether or not the no-go theorem can be circumvented by using a less restrictive ansatz. We should also note that other effective actions for the tachyon dynamics are proposed in the literature (see, e.g., [11, 12]). It would certainly be interesting to check whether or not these actions would allow for a period of positive acceleration which is a pre-requisite for full-SD-brane solutions to exist.

As pointed out before, SD-branes can have different geometries depending on the curvature of the worldvolume and of the transverse space. For example for $k_{||} = 0$ and $k_\perp = 0$ (asymptotically flat) full-SD-branes are such that

$$\begin{aligned} \lim_{t\to\pm\infty} a_{||}(t) &= \text{const.}\,, & \lim_{t\to\pm\infty} \dot{a}_{||}(t) &= 0\,, \\ \lim_{t\to\pm\infty} a_{\perp}(t) &= \text{const.}\,, & \lim_{t\to\pm\infty} \dot{a}_{\perp}(t) &= 0\,. \end{aligned} \tag{2.12}$$

In this case a typical function $a(t)$ is shown on figure 2. Again, this type of behavior can clearly not be achieved unless there is matter in the system allowing a period of constant positive acceleration for the overall volume of the geometry.

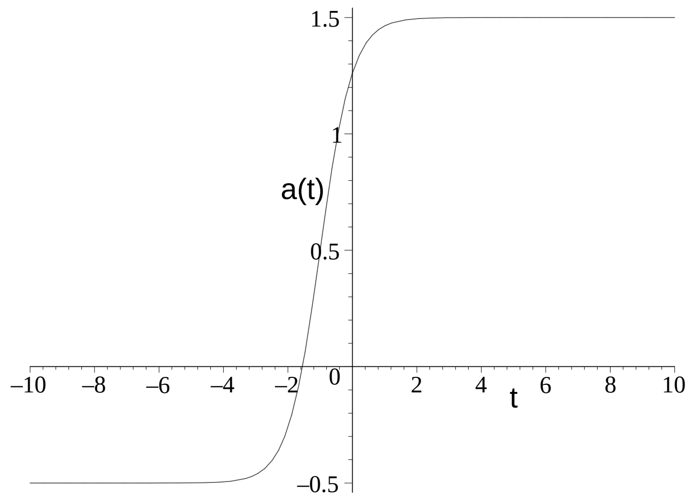

Figure 2: Example of a function $a(t)$ that *would* be associated with a full-SD-brane with $k_{\|} = 0$ and $k_{\perp} = 0$.

Geometric arguments of the type discussed here can clearly be extended to all values of $k_{\|}$ and $k_{\perp}$. This leads to a no-go theorem for homogeneous full-SD-branes ($p \leq 7$) in pure supergravity and in the case where the supergravity modes evolution is driven by an open string tachyon. Note that this is only a reformulation of the singularity theorem of [1]. The latter simply says that that there will always be a curvature singularity (or, more conservatively, a region where $\dot{a}/a$ diverges) in the region $t < 0$ of the type of evolutions illustrated on figures 1 and 2. The geometric no-go theorem was here illustrated for type-I full-SD-branes which are those emerging from a big-bang singularity. Type-II full-SD-branes will roughly be obtained by looking at the $t \to -t$ cases. The no-go theorem applies to those as well. Working from cosmological intuition, we can also be pretty confident that the cases with $k = +1$, for either the parallel or perpendicular scale factors, will do a big-bang or big-crunch anyway, regardless of what type of non-gravitational fields we include.

3. Space-filling SD-branes

In the space-filling case, $p = 8$, the conditions leading to the singularity theorem of [1] are no longer satisfied. It is easy to see this by inspecting the form of I_{tachyon} and I_{sugra} in eq. (2.6). For $p = 8$ the tachyon contribution to the r.h.s. of the equation for $\ddot{a}/a$ is

$$- I_{\text{tachyon}} = +\frac{1}{8}\frac{\lambda V(T) e^{3\Phi/2}}{\sqrt{\Delta}} \left[1 - \frac{9}{2}\dot{T}^2 e^{-\Phi/2}\right] . \tag{3.1}$$

At large times, this is negative. However, the $p = 8$ tachyon contribution to the acceleration is *positive* until $|\dot{T}| = e^{\Phi/4}\sqrt{2}/3$.

For the bulk[2] fields

$$- \frac{1}{9} I_{\text{sugra}} = -\frac{1}{2}\dot{\Phi}^2 + \frac{1}{16} e^{-5\Phi/2} A^2 \, . \tag{3.2}$$

Notice that the contribution of the Ramond-Ramond field is *positive* definite. The metric contribution is absent for $p = 8$.

[2]Of course, for $p = 8$ we can drop the distinction between a and $a_{\|}$ since there is no transverse space.

Therefore, the interesting question to ask is whether there always exists some choice of initial conditions such that the big-crunch can be avoided for a negative initial Hubble parameter. (Note that we expect no singularity trouble for positive initial Hubble parameter; see next sections for details).

Let us consider the constraint carefully. We have for $p = 8$, in Einstein frame,

$$\frac{8 \times 9}{2} \left[H^2 + \frac{k}{a^2} \right] = \rho \,, \tag{3.3}$$

where the energy density is

$$\rho = \frac{\lambda V(T) e^{3\Phi/2}}{2\sqrt{\Delta}} + \frac{1}{4}\dot{\Phi}^2 + \frac{1}{4} e^{-5\Phi/2} A^2 \,. \tag{3.4}$$

Let us consider to the case $k = 0$ for simplicity.[3] Then we have for $t = 0$ the relation $H^2(t = 0) = \rho(t = 0)/36$. So the size of the initial Hubble parameter is set by

$$\left| \frac{\dot{a}(0)}{a(0)} \right| = \frac{1}{6} \sqrt{\rho(0)} \,. \tag{3.5}$$

The question is whether a small enough negative Hubble parameter can be chosen at $t = 0$, consistent with the constraint, such that the positive acceleration at $t = 0$ lifts it up to becoming a positive Hubble parameter before the era of positive acceleration runs out.

The point to notice here is that the answer depends crucially on what the Ramond-Ramond field is doing. On the other hand, the dilaton field cannot help sustain the period of positive acceleration, because as we see above in eq. (3.2), it always contributes negatively to the acceleration. The dilaton also makes the era of positive tachyon contribution to the acceleration shorter. This is because that era ends when $|\dot{T}| = e^{\Phi/4}\sqrt{2}/3$, and in a typical half-SD-brane evolution, the string loop-counting parameter $g_s e^{\Phi}$ falls.

If there are no Ramond-Ramond fields turned on at $t = 0$, then it looks rather unlikely to us that the SD8-brane could remain non-singular with a negative initial Hubble parameter. Let us see why, explicitly. For the purposes of this argument we can turn off the dilaton; as we saw above it only strengthens the likelihood of developing a singularity. So for the moment we are considering a situation with metric and tachyon only. Now, $\rho(0)$ sets the initial Hubble parameter, *and* the initial positive acceleration, *and* whether or not supergravity is actually valid! Explicitly, using eqs. (3.1) and (3.2) we have

$$\frac{\ddot{a}(0)}{a(0)} = \frac{1}{36} \rho(T(0)) \qquad \text{(for } g, T \text{ only)} \,. \tag{3.6}$$

Here, we used the fact that the tachyon derivative is negligible at $t = 0$.

Now, in order for supergravity to be a reasonable approximation to the physics, we require small curvature. Using eq. (2.8) for the Ricci scalar at $t = 0$ we find it to be of order $\rho(T(0))$ for the tachyon contribution, of order $(\dot{a}(0)/a(0))^2$ for the gravity contribution, of order $\dot{\Phi}^2$ and of order $e^{-5\Phi/2}A^2$ for the dilaton and Ramond-Ramond fields respectively. Therefore, each of these quantities, such as $\rho(T(0))$, must be small.

[3] However it should be clear that the case relevant for SD-branes is $k = +1$ since only then will the Friedmann constraint allow the spacetime to bounce between two asymptotically flat regions.

Since for $\rho(0)$ small, $\sqrt{\rho(0)}$ is significantly bigger, it looks unlikely that the era of positive tachyon contribution lasts long enough to turn the Hubble parameter around into positive territory.

On the other hand, this conclusion can change dramatically if we turn on a nonzero Ramond-Ramond field at $t = 0$, because in general

$$\frac{\ddot{a}(0)}{a(0)} = \frac{1}{36}\rho(T(0))\left[1 - \frac{9}{2}\dot{T}^2(0)e^{-5\Phi(0)/2}\right] + \frac{1}{16}e^{-5\Phi/2}A^2(0) - \frac{1}{2}\dot{\Phi}^2 \qquad \text{(general)}\,. \quad (3.7)$$

The Ramond-Ramond contribution to the positive acceleration has to be small, in order for supergravity to remain a decent approximation, but it looks to us that it is quite possible for it to be big enough to lengthen the positive-acceleration period. In addition, although the Ramond-Ramond contribution will typically fall with time in an SD-brane evolution, we can see that it will persist in contributing positively to the acceleration.

The equations are sufficiently complex, even in this restricted ansatz of homogeneous tachyon,[4] that we cannot settle this question absolutely definitively here. However, it looks very likely to us that it is *possible* for the SD8-brane to escape the singularity theorem of [1].

One effect that various approaches to the problem of SD-brane supergravity fields has ignored thus far is particle production. This will typically slow down the rolling tachyon, and therefore potentially prolong the period of positive acceleration. This may help the likelihood of finding a completely non-singular SD8-brane.

We now move to the cases of interest for cosmology.

4. Cosmological applications

Cosmological scenarios involving tachyon condensation have been an area of active investigation since the work of Sen and Gibbons (see, *e.g.*, the review [13]). It is particularly interesting to look at what the results of [1] have to say about these situations, which are of considerable interest to the early universe cosmology community. As we will show, the theorem of [1] simply does not apply; if it did, it would point to a quite general inability to have non-singular cosmologies for matter satisfying physically reasonable energy conditions. In particular, an interesting *non-singular* cosmology involving plain ordinary $d = 4$ Einstein gravity coupled to the tachyon (à la the DBI action) was actually first pointed out in [14].

Let us briefly recap the cosmology story of [14], changing the action to general dimension D (and renormalizing the tachyon potential to conform to the conventions of [1, 10]). We have

$$S = \frac{1}{16\pi G_{D+1}} \int d^{D+1}x\sqrt{-g}\left[\mathcal{R} - \lambda V(T)\sqrt{1 - g^{\mu\nu}\nabla_\mu T\nabla_\nu T}\right]. \quad (4.1)$$

The case of the homogeneous and isotropic rolling tachyon was considered in [14]; subsequently the issues of inhomogeneity were considered in [15].

[4] And, for $p < 8$, branes smeared in the transverse space.

The metric ansatz for the homogeneous case can be taken to be

$$ds^2 = -dt^2 + a^2(t)d\vec{x}_D^2\,, \tag{4.2}$$

where $d\vec{x}_D^2$ is either flat ($k=0$), spherical ($k=+1$) or hyperbolic ($k=-1$). The tachyon equation of motion is not important for us here, except that it rolls fast (nearly at $\dot{T}=1$) at large t.

The constraint equation is

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{1}{D(D-1)}\frac{\lambda V(T)}{\sqrt{1-\dot{T}^2}} \tag{4.3}$$

and can be considered as the initial condition at $t=0$. For $k=0$, $H=\dot{a}/a$ must be nonzero at $t=0$ in order to satisfy the constraint. We can choose either the positive or negative branch (for the initial Hubble parameter) for any given cosmology.

The bulk equation of motion is

$$\frac{\ddot{a}}{a} = \frac{1}{D(D-1)}\frac{\lambda V(T)}{\sqrt{1-\dot{T}^2}}\left[1-\frac{D}{2}\dot{T}^2\right]. \tag{4.4}$$

Notice that this is precisely the same result as we saw in the previous section for the case $p=8$, if the dilaton is negligible.

Following usual cosmological intuition, the *positive* Hubble parameter branch solution of the Friedmann equation is chosen[5] in [14]. Then, according to the bulk equation of motion the scale factor starts out concave up, but as the tachyon gets rolling fast it becomes concave down. But because the tachyon obeys the weak energy condition ($\rho > 0$), $\dot{a}/a$ *always stays positive*. The long-time behaviour is $\ddot{a}/a \sim 0^-$, and $a(t)$ either approaches a *constant* as $t\to\infty$ for $k=0$ or it is *linear in time* for $k=-1$. These give perfectly *non-singular* cosmologies. For $k=+1$, however, there is a big-crunch.

These conclusions hold for any $D>2$, and in particular any dimension D that is sensible for cosmology. All of this is very intuitive, based on previous experience with ordinary minimally coupled scalar fields in four-dimensional cosmology.

4.1 Comments on accelerating cosmology

Although they are singular, the SD2- and SM2-brane spacetimes found in [2, 6, 7] have been studied with the aim of finding characteristics shared with our own universe [16]. The corresponding time-dependent 10- and 11-dimensional spacetimes have been shown to admit, for some values of the parameters in the solutions, periods of positive worldvolume acceleration [16]. There is convincing evidence that the universe is currently in such an accelerating phase. Experimental data also suggest that the dark energy component of the universe has $\Omega_D \sim 70\%$. The papers [16] were considering $\Omega_D = 1$ which is clearly unrealistic. However progress towards achieving more realistic models were made in [17] where additional external sources were added to the supergravity system.

[5]However it is not clear that choosing the negative branch automatically leads to gravitational collapse since the usual cosmological singularity theorems do not apply. This is because the strong energy condition is violated during the evolution of the tachyon.

Certainly, our approach consisting in adding unstable brane sources follows the philosophy of [17]. This approach is in fact a more general study than what was done so far in tachyon cosmology. It would be very interesting to see in details how adding the dilaton and a Ramond-Ramond field would affect the results obtained in tachyon cosmology in a more general way than has been done thus far.

As a first step, we can ask whether the tachyon is likely to prefer positive or negative worldvolume acceleration. As seen before a certain combination of the equations of motion leads to

$$(p+1)\frac{\ddot{a}_{||}}{a_{||}} = -(8-p)\frac{\ddot{a}_{\perp}}{a_{\perp}} - I_{\text{sugra}} - I_{\text{tachyon}}\,. \tag{4.5}$$

It was noted before that for $p = 2$ we have $I_{\text{tachyon}} \geq 0$. Simply by inspection of eq. (4.5) we see that the tachyon contribution to the equations of motion appears to favor negative rather than positive worldvolume acceleration. Since $I_{\text{sugra}} \geq 0$ the only way positive worldvolume acceleration can be achieved is if $\ddot{a}_{\perp}/a_{\perp} < 0$ during the tachyon evolution.

5. Half-SD-branes

In the supergravity approximation, the half-SD-branes, and the unstable D-branes, are morally equivalent to this cosmological case of [14] shown above. In particular, small bulk kicks will be needed at $t = 0$, i.e., we need at least one nonzero (small, positive) Hubble parameter at $t = 0$. For the numerical solutions of [10], the *future* ($t \geq 0$) solutions are not ruled out at all by the singularity theorem of [1].

Let us see this analytically. In the smearing convention of [1], the equations of [10] are easily converted by replacing $\lambda \to \lambda/a_{\perp}^{8-p}$; then we find that $a_{||}(t)$ evolves ever upwards when it starts at $t = 0$ with a positive derivative; $a_{\perp}(t)$ evolves ever upwards as well. In particular, the geometric average scale factor $a(t)$ involved in the singularity theorem of [1] starts out with positive derivative, and it slows down with time (because $\ddot{a} < 0$), eventually asymptoting to a constant. This is clearly consistent with the asymptotics, $a_{||}(t \to \infty) \sim$ const., $a_{\perp}(t \to \infty) \sim t$. This $t \geq 0$ behaviour is also thoroughly consistent intuitively with the results of [14]. As a simple example let us consider the type of evolutions illustrated on figures 1 and 2 where an half-SD-brane corresponds only to the $t \geq 0$ part of the evolution: either the tachyon starts at the top of the potential (perhaps with some initial kinetic energy although the initial conditions $T(0) = 0$ and $\dot{T}(0) = 0$ are consistent as long as there is kinetic energy in the bulk fields at $t = 0$) or the tachyon starts evolving away from the top of the potential. The behaviour shown on the figures is such that $\dot{a}/a \geq 0$ and $\ddot{a}/a \leq 0$ throughout the evolution.

Therefore, we conclude that the $t \geq 0$ parts of the spacetimes of [10] were perfectly legitimate. Let us consider a particular example, i.e., that of an half-SD2-brane. We consider a solution with the boundary conditions[6]

$$\begin{aligned} &T(0) = 0\,, \quad \dot{T}(0) = 0.1\,, \quad a_{||}(0) = 0.1\,, \quad \dot{a}_{||}(0) = 0.079\,, \quad a_{\perp}(0) = 1\,, \\ &\dot{a}_{\perp}(0) = 0\,, \quad \Phi(0) = -1\,, \quad \dot{\Phi}(0) = 0\,, \quad A(0) = 0\,, \end{aligned} \tag{5.1}$$

[6]The graphs are actually displayed with metric components in string frame, for consistency with our previous paper.

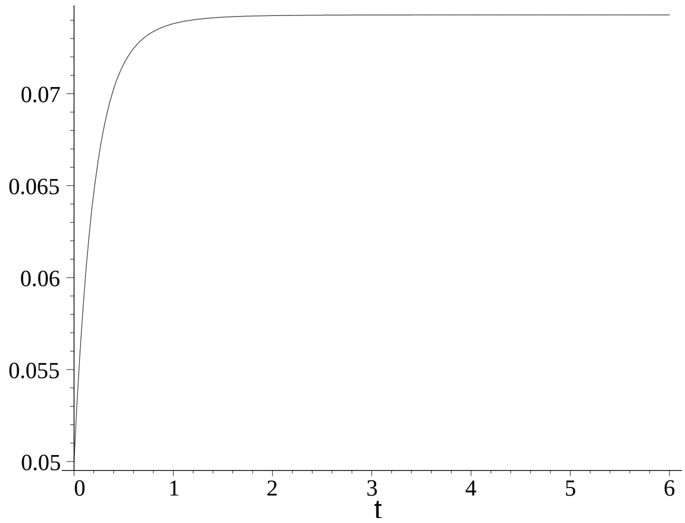

Figure 3: The scale factor on the worldvolume of a supergravity half-SD2-brane with boundary conditions (5.1).

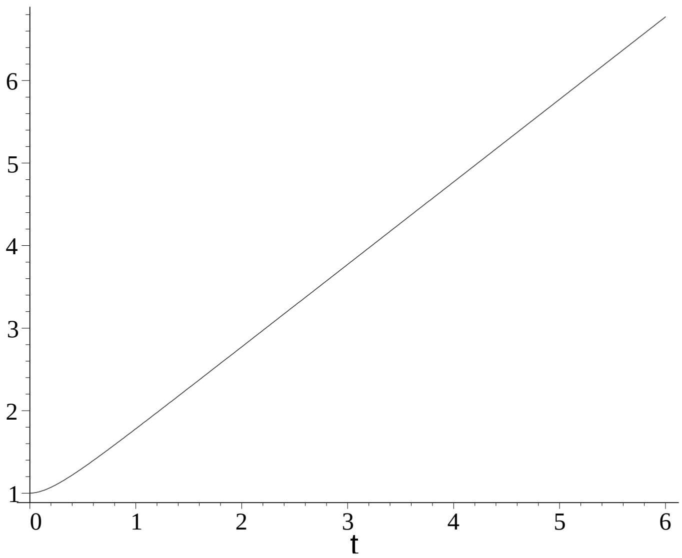

Figure 4: The transverse scale factor $a_\perp(t)$ for a supergravity half-SD2-brane with boundary conditions (5.1).

and the Ramond-Ramond field itself vanishes at $t = 0$. These initial conditions are consistent with the equations of motion including the constraint. Figures 3–7 respectively show the time evolution of the fields $a_{||}(t)$, $a_\perp(t)$, $e^{\Phi(t)}$, the Ramond-Ramond field and $T(t)$. Figure 8 shows the acceleration of the SD-brane volume, i.e., $\ddot{a}/a$.

The singularity theorem of [1] therefore shows that the numerical code of [10] must have been truly unstable — rather than simply exhibiting the flat-space tachyon-matter numerical instability — on the *past* side of the SD-brane evolution. This potential problem was addressed in [10, section 4.4.1] but evidence at the time suggested it was only an

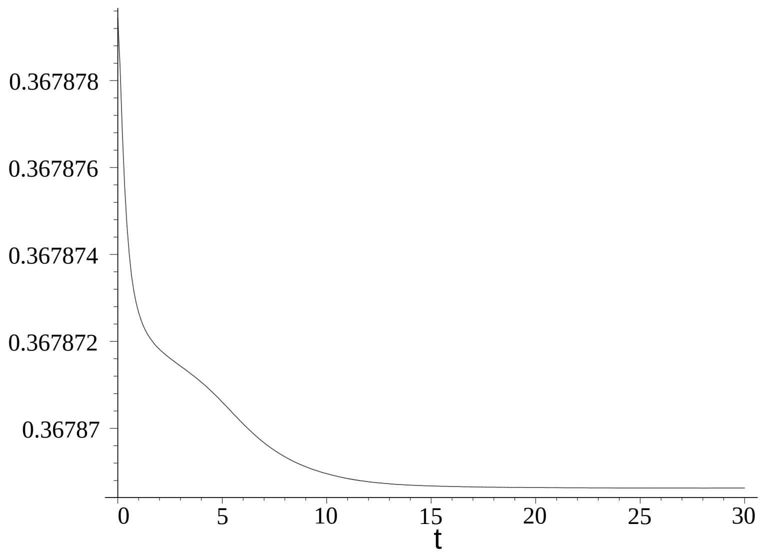

Figure 5: The dilaton function $e^{\Phi(t)}$ for a supergravity half-SD2-brane with boundary conditions (5.1).

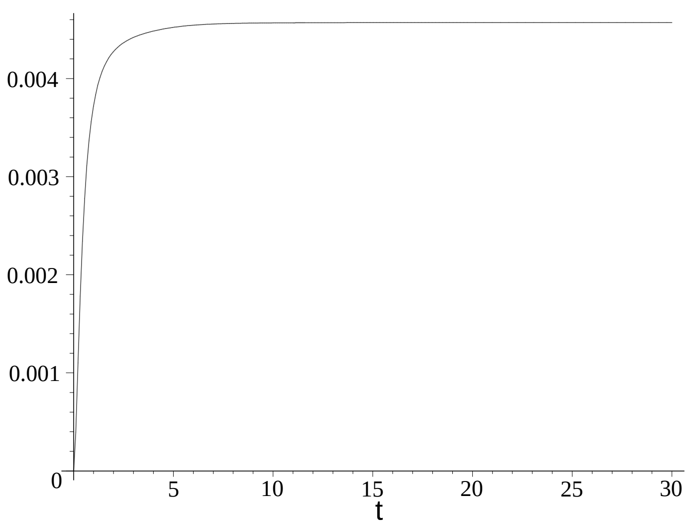

Figure 6: The Ramond-Ramond field for a supergravity half-SD2-brane with boundary conditions (5.1).

artefact of the numerical analysis. An easy way to see this one-sidedness analytically is to consider the easy trick used in [10] to actually evolve numerically from near the tachyon hilltop to $t \to -\infty$: we swap the signs of time and time derivatives of bulk fields at $t = 0$ (and, depending on whether we are considering same-side or opposite-side full SD-branes, we swap the sign of $f(T)$ as well, or not). This means that we are starting the evolution with *negative* Hubble parameter, and since $\ddot{a}/a$ is still negative even with sign-swapped time, we must reach a big-crunch after a finite time in the past of $t = 0$. This conclusion of a past singularity applies whether or not we consider "same-side" or "opposite-side" full SD-branes, i.e., whether the tachyon rolls back to the same minimum or the other minimum.

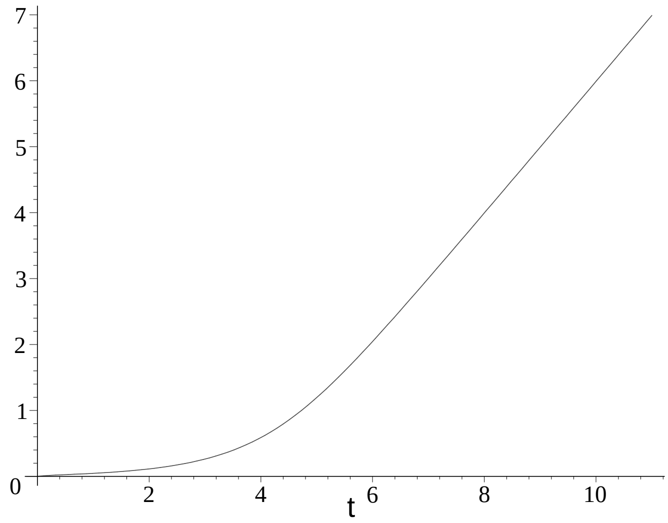

Figure 7: The open string tachyon field associated with a supergravity half-SD2-brane with boundary conditions (5.1).

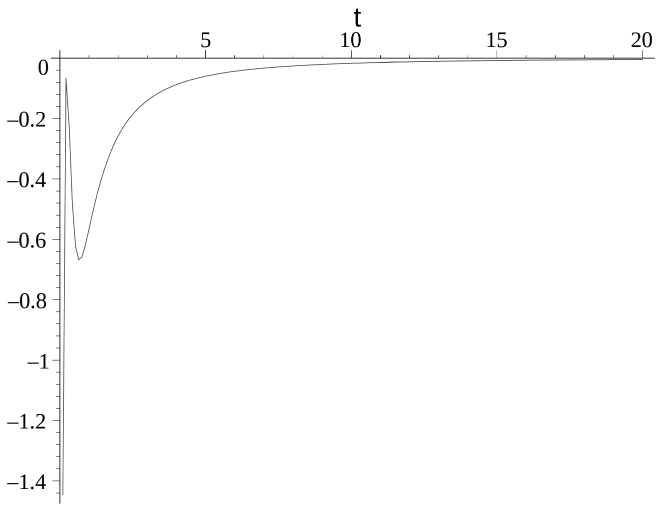

Figure 8: The acceleration $\ddot{a}/a$ for the half-SD2-brane with boundary conditions (5.1).

6. Discussion

In this short note, we have shown in precise details with the aid of the new analytic results of [1] where the numerical results of our previous work [10] broke down. We have also pointed out that cosmology with rolling tachyons is of course not ruled out. In particular, half-SD-branes certainly exist as well-behaved supergravity solutions, provided that small (positive) bulk kicks are given to at least one bulk field at $t = 0$, such as $a_{||}(t)$. As we saw in the cosmology section 4, this initial condition choice is in tune with textbook cosmological intuition.

Most interesting directions for future work are manyfold. We have already pointed out here (in section 2) that anisotropy on the worldvolume may provide a way of escaping the singularity theorem of [1].

The general question of inhomogeneities in tachyon cosmology deserves further investigation. In particular, in the current context it will be interesting to know whether the singularity theorem of [1] can be extended to cover those cases. Intuition based on standard cosmology indicates that inhomogeneity will probably only strengthen the no-go theorem, but this must be checked, since inhomogeneities notably complicate all the field equations.

Another thing to check will be the role of *un*smearing the branes in the transverse space. This is important because the validity of the no-go theorem is limited by the restrictive nature of the supergravity ansatz used in deriving it. We feel this issue has particular importance because we have pointed out that the SD8-brane may actually avoid the no-go theorem. And the $p = 8$ case is really the only one "honestly" covered by the ansatz used.

It could be interesting to include in this investigation also the question of whether changes in the low-energy effective action for the tachyon, as suggested in [11], may change the outcome.

The details of how the tachyon rolls can be significantly affected by particle production when backreaction is included. In particular, if the rolling slows down because of particle production, there may be a longer time of positive acceleration for the space-filling case.

Finally, there is the question of whether quantum string theoretic ingredients are unavoidably necessary for description of SD-branes. In [1] particular emphasis was put on the fact that the nonexistence of SD-brane supergravity solutions — established via the no-go theorem — is in tune with their picture that SD-brane decay should be essentially quantum-mechanical. We think that this issue depends crucially on whether the space-filling SD-branes escape the no-go theorem or not. If they do, then it would seem rather odd that whether or not quantum stringy ingredients are needed depends so crucially on the codimension.

It is possible that quantum stringy ingredients may turn out to be important for getting long periods of positive acceleration in models of this type. Clearly, an important issue is whether supergravity can remain a decent approximation to the physics in such a situation. Let us suppose for the sake of argument that this can be done. The program of [8, 10] consists in solving the Einstein equations with sources in the form of a perfect fluid with stress-energy tensor components

$$\rho = \frac{\lambda V(T) e^{\phi(p/4)-1/2}}{2a_\perp^{8-p}\sqrt{\Delta}} + \frac{1}{4}\dot{\Phi}^2 + \frac{1}{4}e^{a\Phi}A^2 \,, \tag{6.1}$$

$$P_\parallel = -\frac{\lambda V(T) e^{\phi(p/4)-1/2}\sqrt{\Delta}}{2a_\perp^{8-p}} + \frac{1}{4}\dot{\Phi}^2 - \frac{1}{4}e^{a\Phi}A^2 \,, \tag{6.2}$$

$$P_\perp = \frac{1}{4}\dot{\Phi}^2 + \frac{1}{4}e^{a\Phi}A^2 \,. \tag{6.3}$$

Each of these sources separately satisfies the dominant energy condition (see [18] for definitions). Of course, this implies that the weak energy condition and the null convergence condition are satisfied as well. It was noted in [1] that the strong energy condition,

$$7\rho + (p+1)P_\parallel + (8-p)P_\perp \geq 0 \,, \tag{6.4}$$

is satisfied by eqs. (6.1)–(6.3) for $p < 7$. As shown in section 2, circumventing the no-go theorem would, as a minimum requirement, require that a period of positive acceleration (for the volume V_{S} of the spacetime) be allowed. This could be achieved by introducing new terms (i.e., sources with a different equation of state) to the r.h.s. of (2.4) that contribute positively, i.e., in such a way as to violate the strong energy condition. Sources in the form of massive open or closed strings might achieve the desired behavior therefore resolving the singularity and circumventing the no-go theorem. Analysis of massive open string mode production during tachyon evolution were performed in [3]–[5]. BCFT calculations for the emission of massive closed strings can be found in [19, 20].

To illustrate our point let us use a simple example, i.e., that of a scalar field in four dimensions, with the usual with stress-energy tensor. An implication of the strong energy condition being satisfied is that

$$\left(T_{ab} - \frac{1}{2}g_{ab}T^c_c\right)\xi^a\xi^b \geq 0\,, \tag{6.5}$$

which becomes

$$(\nabla_a\phi\,\xi^a)^2 - \frac{1}{2}m^2\phi^2 \geq 0 \tag{6.6}$$

for a massive scalar field. ξ^a is a timelike unit vector. After using the equation of motion and integrating over a region M of the spacetime the l.h.s. of (6.6) becomes

$$\frac{1}{2}\int_M \left(g^{ab} + 2\xi^a\xi^b\right)\nabla_a\phi\nabla_b\phi\, d^4x - \frac{1}{2}\int_{\partial M}\phi\nabla_a g^{ab}dn_b\,, \tag{6.7}$$

where ∂M is a boundary and dn_b a normal vector. The first term in eq. (6.7) is always positive while the second term, which is negative-definite, will remain small as long as the region of integration M is large compare to the Compton wavelength ($\lambda = 1/m$) of the mode. The strong energy condition might therefore be violated in small regions of the spacetime in particular when the domain of existence (in time and space) of the modes is small. This simple line of reasoning suggest that the emission of massive modes (open and/or closed) might provide a loophole to the singularity theorem for full-S-branes. Of course,as we mentioned above, this assumes that the tachyon evolution, including emission of massless and massive open and closed strings is smooth and that the contribution of the massive modes does not overwhelm that of the massless modes.

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